

Non-abelian Action for Multiple Five-Branes with Self-Dual Tensors

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ABSTRACT: We construct an action for non-abelian 2-form in 6-dimensions. Our action consists of a non-abelian generalization of the abelian action of Perry and Schwarz for a single five-brane. It admits a self-duality equation on the field strength as the equation of motion. It has a modified 6d Lorentz symmetry. On dimensional reduction on a circle, our action gives the standard 5d Yang-Mills action plus higher order corrections. Based on these properties, we propose that our theory describes the gauge sector of multiple M5-branes in flat space.

KEYWORDS: M-Theory, D-branes, M-branes, Gauge Symmetry.

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1. Introduction

The low energy theory of N coincident M5-branes is given by an interacting (2,0) superconformal theory in 6 dimensions [1]. For a single M5-brane, the low energy theory is known [2–6]. So far very little is known about this theory for $N > 1$. There are a number of difficulties associated with this theory. First, the structure of (2,0) supersymmetry constraints the 2-form potential to have self-dual field strength. This makes it difficult to write down a Lorentz invariant action. This problem was solved in [3–5] where an action principle was constructed with the self-duality equation obtained as the equation of motion. For the non-abelian case, there is an additional problem that an appropriate generalization of the tensor gauge symmetry was not known. In particular, there are no-go theorems [7] which state that there is no nontrivial deformation of the Abelian

2-form gauge theory if locality of the action and the transformation laws are assumed. The no-go theorems suggest an important direction to go is to give up locality.

Since M2-branes can end on M5-branes, one may wonder what one may learn by considering the intersecting M2-M5 branes system. In the paper [8], a system of open N M2-branes described by the open ABJM theory [9] is considered. The gauge non-invariance of the boundary Chern-Simons action was shown to imply the existence of a Kac-Moody current algebra on the worldsheet of multiple self-dual strings. It was conjectured [10] that the Kac-Moody symmetry induces a $U(N) \times U(N)$ gauge symmetry in the theory of N coincident M5-branes. The precise nature of this gauge symmetry in the theory of M5-branes is however not known due to our little understanding of the self-dual strings. Motivated by this, in [10] a set of $U(N) \times U(N)$ gauge bosons was introduced and a version of non-abelian generalization of the tensor gauge symmetry of 2-form gauge potentials was constructed. This formulation has the advantage of having manifest Lorentz symmetry fully.

Generally, the non-abelian tensor gauge symmetry is linearly represented if the $U(N) \times U(N)$ gauge bosons are treated as independent fields. On the other hand, the (2,0) supersymmetry of M5-branes implies that no extra degrees of freedom is allowed and so these fields must be taken as auxiliary. This turns out to be very difficult for one of the auxiliary fields. So in this paper we will consider a gauge fixed approach by given up manifest 6d Lorentz symmetry.

As a first step towards understanding the theory of multiple M5-branes, we will focus on the chiral tensor gauge fields in this paper. Our action consists of a non-abelian generalization of the action of Perry and Schwarz [3] plus an additional term which sets the Yang-Mills gauge fields to become auxiliary. We emphasize that the action of Perry-Schwarz (PS) is of the same type as the action originally introduced by Henneaux and Teitelboim (HT) [11], see also [12] for a recent discussion. The difference is that a time direction was separated from the rest in HT action as they were interested in a Hamiltonian description, while in the PS action a space direction was separated from the (5+1) dimensional spacetime, making it particularly suitable for discussing dimensional reduction of the system¹. Since we will be interested in dimensional reduction of our action, so we will follow [3] in this paper. As in Perry-Schwarz's construction, a direction x_5 is singled out and specially treated, so our theory is only manifestly 5d Lorentz invariant. Nevertheless, we manage to establish the existence of an additional non-manifest 6d Lorentz symmetry, generalizing the result of the abelian case [3, 11]. Moreover, on dimensional reduction on a circle, our action gives rise directly to the standard 5d Yang-Mills theory plus higher order corrections. Based on these properties, we propose that our action describes the gauge sector of a system of coincident M5-branes in

¹The covariant Pasti-Sorokin-Tonin (PST) formulation [5] unifies both since one can gauge fix the auxiliary scalar to arrive at these different formulations.

flat space. The tensor gauge symmetry in our action turns out to be abelian, but highly nonlinear and nonlocal. In fact whether the tensor gauge symmetry is abelian or non-abelian is not constrained by any physical requirement we know of. The abelian nature of the tensor gauge symmetry is thus a prediction of our construction. The construction of a non-abelian tensor gauge symmetry is still an interesting mathematical question, but from our construction it seems not necessary for the non-covariant description of multiple M5-branes.

The plan of the paper is as follows. In section 2, we review the construction of Perry and Schwarz [3]. In section 3, we present our construction of the action for non-abelian 2-form fields and establish the properties of self-duality, 6d Lorentz symmetry and dimensional reduction to 5d Yang-Mills action. Section 4 contains some further discussions. In particular we comment on the inclusion of fermions and scalar fields and supersymmetry in the discussion section. For completeness, three appendices are included which treat some analysis in the main text in more details.

Recent related works on the subject includes: [13, 14] which proposed a fundamental definition of multiple M5-branes in terms of 5d supersymmetric Yang-Mills theory; [15] which constructed a non-abelian version of (2,0) supersymmetric equation of motion using Lie 3-algebra; [16] which constructed a compactified theory of non-abelian 2-form gauge potentials with a self-dual field strength; [17] which proposed a more general framework than [10] in utilizing a 3-form gauge potentials in addition to the 1-form gauge potentials; [18–20] which studied the form of quantum geometry of M5-branes in a C -field background; [21] on amplitudes of multiple M5-branes theory; [22] on the N^3 entropy counting of M5-branes; as well as other issues concerning multiple M5-branes [23]. For a review on older results on M5-branes and superconformal theory in 6-dimensions, we suggest [24].

2. Abelian Action of Perry-Schwarz

Let us start by reviewing the construction [3, 11] of an action for a self-dual tensor in 6-dimensions. A key feature of their construction is that a certain direction, x^0 in [11] or x^5 in [3], has to be singled out and so the formulation has only manifestly 5d rotational invariance or 5d Lorentz invariance. Nevertheless these theories do possess the full Lorentz symmetry. The existence of this modified Lorentz symmetry is a remarkable feature of these constructions.

We will be interested in the Lagrangian formulation of the chiral tensor gauge fields on multiple M5-branes and its dimensional reduction. Therefore let us follow the construction of Perry-Schwarz [3] in the following. Let us denote the 5d and 6d coordinates by $x^\mu = (x^0, x^1, \dots, x^4)$ and $x^M = (x^\mu, x^5)$. We adopt the convention

$\eta^{MN} = (- + + + +)$ for the metric and

$$\epsilon^{01234} = -\epsilon_{01234} = 1, \quad \epsilon^{012345} = -\epsilon_{012345} = 1 \quad (2.1)$$

for the antisymmetric tensors. The Hodge dual of a 3-form G_{MNP} is defined by

$$\tilde{G}_{MNP} := -\frac{1}{6}\epsilon_{MNPQRS}G^{QRS}. \quad (2.2)$$

Note the minus sign in our definition of the Hodge dual follows from our convention of the antisymmetric tensor (2.1) which says that the 6d orientation is specified by $dx^0dx^1 \cdots dx^5$. The abelian field strength is given by

$$H_{MNP} = \partial_M B_{NP} + \partial_N B_{PM} + \partial_P B_{MN} := \partial_{[M} B_{NP]} \quad (2.3)$$

and the self-duality equation reads

$$\tilde{H}_{MNP} = H_{MNP}. \quad (2.4)$$

In the Perry-Schwarz formulation, the self-dual tensor gauge field is represented by a 5×5 antisymmetric tensor field $B_{\mu\nu}$. The action reads

$$S_0(B) = \frac{1}{2} \int d^6x \left(-\tilde{H}^{\mu\nu}\tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu}\partial_5 B_{\mu\nu} \right) \quad (2.5)$$

where

$$\tilde{H}^{\mu\nu} := \frac{1}{6}\epsilon^{\mu\nu\rho\lambda\sigma}H_{\rho\lambda\sigma}, \quad H^{\mu\nu\rho} = -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda\sigma}\tilde{H}_{\lambda\sigma}. \quad (2.6)$$

The action has the second order equation of motion

$$\epsilon^{\mu\nu\rho\lambda\sigma}\partial_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0 \quad (2.7)$$

which has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \Phi_{\lambda\sigma} \quad (2.8)$$

for some function $\Phi_{\lambda\sigma}$ such that $\partial_{[\mu}\Phi_{\lambda\sigma]} = 0$. It is easy to check that the action (2.5) is invariant ² under the gauge symmetry

$$\delta B_{\mu\nu} = \Sigma_{\mu\nu} \quad (2.9)$$

for arbitrary $\Sigma_{\mu\nu}$ such that $\partial_{[\mu}\Sigma_{\nu\lambda]} = 0$, or equivalently

$$\delta B_{\mu\nu} = \partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu, \quad \text{for arbitrary } \varphi_\mu. \quad (2.10)$$

²This is under the usual assumption that fields, in this case $H_{\mu\nu\lambda}$, vanishes at infinity $|x^\mu| = \infty$.

This is the tensor gauge symmetry of the model. An appropriate gauge fixing of this symmetry allows one to reduce the general solution (2.8) to the special form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}. \quad (2.11)$$

This is the self-duality equation in this theory.

The action is manifestly 5d Lorentz invariant. Nevertheless the action is indeed invariant under an additional Lorentz transformation mixing the μ directions with the 5 direction. The proposed modified Lorentz transformation is

$$\delta B_{\mu\nu} = (\Lambda \cdot x) \tilde{H}_{\mu\nu} - x_5 (\Lambda \cdot \partial) B_{\mu\nu}, \quad (2.12)$$

where $\Lambda_\mu = \Lambda_{5\mu}$ denote the corresponding infinitesimal transformation parameters. One can check that

$$[\delta_{\Lambda_1}, \delta_{\Lambda_2}] B_{\mu\nu} = \delta_{\Lambda_{\alpha\beta}}^{(5d)} B_{\mu\nu} + \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu \quad (2.13)$$

gives, apart from terms that vanish on-shell (2.11), the expected 5d Lorentz transformation

$$\delta_{\Lambda_{\alpha\beta}}^{(5d)} B_{\mu\nu} = \Lambda_\mu^\lambda B_{\lambda\nu} - \Lambda_\nu^\lambda B_{\lambda\mu} + x_\lambda \Lambda^{\lambda\alpha} \partial_\alpha B_{\mu\nu} \quad (2.14)$$

plus the gauge transformation (2.10). The parameters are

$$\Lambda_{\mu\nu} = \Lambda_{1\mu} \Lambda_{2\nu} - \Lambda_{1\nu} \Lambda_{2\mu}, \quad \varphi_\nu = x^\alpha \Lambda_{\alpha\lambda} B_\nu^\lambda. \quad (2.15)$$

Therefore the modified Lorentz transformation (2.12) does give rise to the desired 6d Lorentz group.

A couple of remarks follow concerning the Perry-Schwarz construction.

1. We note that in the proof [3] of the invariance of the action (2.5) under the Lorentz transformation (2.12), various total derivatives terms in the variation of the action were dropped under the natural assumption that

$$\partial_\lambda B_{\mu\nu} \rightarrow 0 \text{ as } |x^M| \rightarrow \infty. \quad (2.16)$$

Under the same assumption, the self-duality equation of motion (2.11) holds since $H_{\mu\nu\lambda} \rightarrow 0$ at infinity.

2. The Perry-Schwarz theory is based on the set of fields $B_{\mu\nu}$ which nevertheless is 6d Lorentz invariant. That it is possible to support the Lorentz symmetry without introducing the components $B_{\mu 5}$ is entirely due to the existence of the gauge symmetry (2.10) in the theory. In the manifestly Lorentz covariant formulation of PST [5], the field $B_{\mu\nu}$ is extended to B_{MN} . In addition an auxiliary scalar field a is introduced with new gauge symmetries that allow one to choose the gauge $B_{\mu 5} = 0$ and $a = x_5$. In this gauge, the Perry-Schwarz action is obtained.

3. One may also combine the modified Lorentz transformation (2.12) with the gauge transformation (2.10) with a parameter $\varphi_\mu = -x_5 B_{\mu\kappa} \Lambda^\kappa$ and obtain an equivalent form of the modified Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x) \tilde{H}_{\mu\nu} - x_5 \Lambda^\kappa H_{\kappa\mu\nu}, \quad (2.17)$$

which is written entirely in terms of the field strength. The check of the invariance of the action under (2.17) is included in the appendix.

3. Action for Non-Abelian Self-Dual Two-Form on M5-Branes

For simplicity, we will construct a theory of the 2-form potential without scalars and fermions. Supersymmetry is important and will be considered separately. For the gauge part, motivated by the construction of [10], we consider the addition of a set of 1-form gauge fields A_M^a for a gauge group G .

3.1 Non-Abelian action

Following the above discussion, we will give up manifest 6d Lorentz symmetry and represent the self-dual tensor gauge field by a 5×5 antisymmetric field $B_{\mu\nu}$ in the adjoint. Since there is no room for extra degrees of freedom in the (2,0) tensor multiplets of M5-branes, therefore the gauge fields A_M must be determined in terms of the tensor gauge fields. It turns out we need to take the Yang-Mills gauge field to be a 5-dimensional field living in the 5d space x^μ , i.e. $A_\mu = A_\mu(x^\lambda)$ ³. Let us introduce the following non-abelian generalization of the Perry-Schwarz action

$$S_0 = \frac{1}{2} \int d^6x \text{tr} \left(-\tilde{H}^{\mu\nu} \tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu} \partial_5 B_{\mu\nu} \right), \quad (3.1)$$

where

$$H_{\mu\nu\lambda} = D_\mu B_{\nu\lambda} + D_\nu B_{\lambda\mu} + D_\lambda B_{\mu\nu} \quad (3.2)$$

and

$$\tilde{H}^{\mu\nu} = \frac{1}{6} \epsilon^{\mu\nu\rho\lambda\sigma} H_{\rho\lambda\sigma} \quad (3.3)$$

is the Hodge dual of $H_{\mu\nu\lambda}$. $H_{\mu\nu\lambda}$ obeys the modified Bianchi identity

$$D_{[\mu} H_{\nu\lambda\rho]} = \frac{3}{2} [F_{[\mu\nu}, B_{\lambda\rho]}]. \quad (3.4)$$

³We note that a 5-dimensional gauge field was also employed in [16]. However our construction differs from theirs in essential ways: a compactified spacetime was considered in [16] and the gauge field was taken to be the zero mode of the tensor gauge field $B_{\mu 5}^{(0)}$. In our construction, we do not compactify the spacetime and A_μ is given by an integrated expression (3.12) on shell. We thank Pei-Ming Ho for a discussion on this point

The action S_0 is invariant under the Yang-Mills gauge symmetry

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \text{for arbitrary } \Lambda = \Lambda(x^\lambda), \quad (3.5)$$

$$\delta B_{\mu\nu} = [B_{\mu\nu}, \Lambda], \quad \delta H_{\mu\nu\lambda} = [H_{\mu\nu\lambda}, \Lambda] \quad (3.6)$$

and the following “tensor gauge symmetry” ⁴:

$$\delta_T A_\mu = 0, \quad (3.8)$$

$$\delta_T B_{\mu\nu} = \Sigma_{\mu\nu}, \quad \text{for arbitrary } \Sigma_{\mu\nu}(x^M) \text{ such that } D_{[\lambda} \Sigma_{\mu\nu]} = 0. \quad (3.9)$$

It is $[\delta_{T(1)}, \delta_{T(2)}] = 0$ and so the tensor gauge symmetry is abelian. Like the abelian case, we will consider field configurations with vanishing covariant derivatives at infinity:

$$D_\lambda B_{\mu\nu}, \quad \partial_5 B_{\mu\nu} \rightarrow 0 \quad \text{as } |x^M| \rightarrow \infty. \quad (3.10)$$

It follows that $H_{\mu\nu\lambda}$ vanishes at infinity also.

An important observation is that the condition for the vanishing of field strength at infinity:

$$H_{\mu\nu\lambda} \rightarrow 0, \quad \text{at } x_5 \rightarrow \pm\infty \quad (3.11)$$

is equivalent to the Bianchi identity of the gauge field A_μ if $F_{\mu\nu}$ is identified with the boundary value of $B_{\mu\nu}$, e.g. $F_{\mu\nu} = B_{\mu\nu}(x_5 = \infty)$. With the anticipation of the self-duality equation of motion (3.27) in our theory, we will consider a different constraint

$$F_{\mu\nu} = \int dx_5 \tilde{H}_{\mu\nu}. \quad (3.12)$$

With the constraint (3.12), there is no new degrees of freedom carried by A_μ ⁵. We will implement (3.12) in the action by introducing a 5-dimensional auxiliary field $E_{\mu\nu}(x^\mu)$ and add the action

$$S_E = \int d^5x \text{tr} \left((F_{\mu\nu} - \int dx_5 \tilde{H}_{\mu\nu}) E^{\mu\nu} \right). \quad (3.13)$$

⁴Or equivalently

$$\delta_T B_{\mu\nu} = D_\mu \Lambda_\nu - D_\nu \Lambda_\mu \quad \text{for arbitrary } \Lambda_\mu(x^M) \text{ such that } [F_{[\mu\nu}, \Lambda_{\lambda]}] = 0. \quad (3.7)$$

⁵One may be tempted to use a Chern-Simons action to enforce the gauge field to be auxiliary. However unlike the 3-dimensional case where a Chern-Simons gauge field is auxiliary and contains no local degrees of freedom, pure Chern-Simons gauge field in 5-dimension contains local degrees of freedom [25–27]. In the appendix, we review this argument as well as the extension for Chern-Simons coupled to a conserved source.

The boundary condition of $E_{\mu\nu}$ will be taken as the trivial one

$$E_{\mu\nu} \rightarrow 0 \quad \text{as} \quad |x^\lambda| \rightarrow \infty. \quad (3.14)$$

$E_{\mu\nu}$ transforms under Yang-Mills and tensor gauge transformation as

$$\delta E_{\mu\nu} = [E_{\mu\nu}, \Lambda], \quad \delta_T E_{\mu\nu} = 0 \quad (3.15)$$

and so S_E is invariant. The action is also invariant under the gauge symmetry

$$\delta E_{\mu\nu} = \alpha_{\mu\nu} \quad (3.16)$$

for arbitrary $\alpha(x^\lambda)$ such that

$$D_{[\mu} \alpha_{\nu\lambda]} = 0, \quad D^\mu \alpha_{\mu\lambda} = 0, \quad \text{and} \quad \alpha \rightarrow 0 \text{ as } |x^\lambda| \rightarrow \infty. \quad (3.17)$$

All in all, we propose the following action for a non-abelian theory of self-dual tensor

$$S = S_0 + S_E. \quad (3.18)$$

The action S is Yang-Mills gauge invariant and tensor gauge invariant. It is also invariant under the gauge symmetry (3.16) of $E_{\mu\nu}$. Five dimensional Lorentz symmetry is manifest. We will show below this action leads to a self-duality equation of motion. We will also demonstrate the existence of a non-manifest 6d Lorentz symmetry in our theory and the connection to 5d Yang-Mills theory of multiple D4-branes through dimensional reduction on a circle. The form of the constraint (3.12) is inspired by the analysis of this reduction.

3.2 Properties

3.2.1 Self-duality

The equation of motion of $E_{\mu\nu}$ gives the constraint

$$F_{\mu\nu} = \int dx_5 \tilde{H}_{\mu\nu}. \quad (3.19)$$

This has to satisfy the Bianchi identity

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho F_{\lambda\sigma} = 0. \quad (3.20)$$

For $B_{\mu\nu}$, we have

$$\delta S_0 = \frac{1}{2} \int \epsilon^{\mu\nu\rho\lambda\sigma} \delta B_{\mu\nu} D_\rho (H_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) \quad (3.21)$$

and hence the equation of motion

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho (\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} + E_{\lambda\sigma}) = 0, \quad (3.22)$$

Integrating it over x_5 , we get

$$D_{[\rho} E_{\lambda\sigma]} = 0. \quad (3.23)$$

In fact $\int dx_5 \epsilon^{\mu\nu\rho\lambda\sigma} D_\rho (\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$ where we have used (3.19) and the Bianchi identity of $F_{\mu\nu}$, and we have assumed that $H_{\mu\nu\lambda}$ vanishes at $x^5 = \pm\infty$. Our claim follows from the fact that $E_{\lambda\sigma}$ is independent of x_5 . As a result, the equation (3.22) reads

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho (\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0 \quad (3.24)$$

and has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \Phi_{\lambda\sigma}, \quad (3.25)$$

where

$$D_{[\lambda} \Phi_{\mu\nu]} = 0. \quad (3.26)$$

Therefore with an appropriate fixing of the gauge symmetry (3.9), one can always reduce the second order equation (3.25) to the first order form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}. \quad (3.27)$$

This is the form of the self-duality equation in our theory.

The equation (3.27) implies that on-shell, $F_{\mu\nu}$ is simply given in terms of the boundary values of $B_{\mu\nu}$:

$$F_{\mu\nu} = B_{\mu\nu}(x_5 = \infty) - B_{\mu\nu}(x_5 = -\infty), \quad (3.28)$$

and Bianchi identity is satisfied since the field strength vanishes at infinity. Finally, the equation of motion for A_μ gives

$$D^\mu E_{\mu\nu} - \frac{1}{4} \int dx_5 \epsilon_\nu^{\alpha\beta\gamma\delta} [B_{\alpha\beta}, E_{\gamma\delta}] = -\frac{1}{2} \int dx_5 \epsilon_\nu^{\alpha\beta\gamma\delta} [B_{\alpha\beta}, \partial_5 B_{\gamma\delta} - \frac{1}{2} \tilde{H}_{\gamma\delta}] := J^\nu. \quad (3.29)$$

We note that as a result of the self-duality equation of motion (3.27), the “current” is covariantly conserved $D_\lambda J^\lambda = 0$. Of course (3.29) is consistent with this.

Summarizing, the equations of motion in our theory are the auxiliary equation for A_μ (3.12), the self-duality equation (3.27) and the equations (3.23) and (3.29) for $E_{\mu\nu}$. Note that on eliminating A_μ using (3.12), the self-duality equation (3.27) is self-interacting and is completely independent of $E_{\mu\nu}$.

The counting of the degrees of freedom in our theory goes as follows. The equation of motion (3.19) says A_μ is auxiliary and is determined entirely in terms of $\tilde{H}_{\mu\nu}$. Using this, the action S can be written as a nonlocal action in terms of expansion in powers of $B_{\mu\nu}$. At the quadratic level, the action is simply given by $\dim G$ copies of the Perry-Schwarz action, plus the action S_E . For small field strengths, we can take the higher order terms as small corrections and we can count the degrees of freedom using the linearized theory. In this limit, $A_\mu = 0$ and the tensor gauge symmetry and the self-duality equation of

motion are precisely those of the original Perry-Schwarz theory. Thus we obtain $3 \times \dim G$ degrees of freedom in $B_{\mu\nu}$. As for $E_{\mu\nu}$, the linearized equations of motion are

$$\partial_{[\mu} E_{\nu\lambda]} = 0, \quad \partial^\mu E_{\mu\nu} = 0, \quad (3.30)$$

and there is the gauge symmetry (3.16) with the parameters $\alpha_{\mu\nu}$ satisfying, in this case,

$$\partial_{[\mu} \alpha_{\nu\lambda]} = 0, \quad \partial^\mu \alpha_{\mu\nu} = 0. \quad (3.31)$$

Since $E_{\mu\nu}$ and $\alpha_{\mu\nu}$ also satisfy the same (vanishing) boundary condition at infinity, so we can use the gauge symmetry to remove the $E_{\mu\nu}$ field completely. This is compatible with the fact $E_{\mu\nu}$ was introduced as an auxiliary field to implement the constraint (3.12). All in all, our theory contains $3 \times \dim G$ degrees of freedom as required by (2,0) supersymmetry

We remark that when $B_{\mu\nu}$ is diagonal with distinct diagonal elements such that the gauge group is broken down to $U(1)^r$ (r is the rank of the gauge group), our action reduces to a sum of r copies of the abelian Perry-Schwarz theory and describes the gauge sector of r separated M5-branes. More generally, once the scalar and fermion fields are included in the theory, one can have a system of lumps of coincident M5-branes, BPS or non-BPS relative to each other; and as usual, the pattern of symmetry breaking as well as the interacting dynamics of M5-branes can be studied.

3.2.2 Lorentz symmetry

Our action is manifestly 5d Lorentz invariant. It is straightforward to check that it is not invariant under the modified Lorentz transformation (2.12) or (2.17). See appendix A for the check. Let us proceed by further modifying the Lorentz transformation. We observe that the equation (3.21) for the variation of S_0 under a general variation of $\delta B_{\mu\nu}$ can be rewritten as

$$\delta S_0 = \int d^6x \operatorname{tr} \left[\Delta B^{\mu\nu} \tilde{H}_{\mu\nu} \right], \quad (3.32)$$

where

$$\Delta B^{\mu\nu} := \partial_5(\delta B^{\mu\nu}) - \frac{1}{2}\epsilon^{\mu\nu\alpha\beta\gamma}D_\alpha(\delta B_{\beta\gamma}). \quad (3.33)$$

It is interesting to note that

$$\Delta B_{\mu\nu} = -\delta(\tilde{H}_{\mu\nu} - \partial_5 B_{\mu\nu}), \quad (3.34)$$

which is just the variation of the self-duality equation of motion.

Taking $\delta B_{\mu\nu}$ now as the 5- μ Lorentz transformation, it is clear that the action will be invariant if the variation satisfies $\Delta B_{\mu\nu} = 0$. This is a sufficient condition, but not necessary. In fact $\Delta B_{\mu\nu} \neq 0$ for the abelian case (2.17), nevertheless S_0 is invariant. So let us consider a general transformation of the form

$$\delta B_{\mu\nu} = (\Lambda \cdot x) \tilde{H}_{\mu\nu} - \lambda x_5 \Lambda^\kappa H_{\kappa\mu\nu} + \Lambda^\kappa \phi_{\mu\nu\kappa} := \delta_{(1)} B_{\mu\nu} + \delta_{(2)} B_{\mu\nu}, \quad (3.35)$$

where λ is a constant and $\phi_{\mu\nu\kappa} = -\phi_{\nu\mu\kappa}$ is a quantity to be determined by demanding S_0 to be invariant. We have denoted the first two variation terms by $\delta_{(1)}B_{\mu\nu}$ and the third term by $\delta_{(2)}B_{\mu\nu}$. By redefining $\phi_{\mu\nu\kappa}$ with an appropriate shift, one can bring λ to any value one wants. This freedom will turn out to be convenient.

The variation of S_0 under $\delta_{(1)}B_{\mu\nu}$ is

$$\delta_{(1)}S_0 = \int \left[\frac{\lambda}{2}x_5\epsilon^{\mu\nu\alpha\beta\gamma}D_\alpha H_{\beta\gamma\kappa}\Lambda^\kappa + \frac{\lambda-1}{4}\Lambda_\rho\tilde{H}_{\alpha\beta}\epsilon^{\rho\alpha\beta\mu\nu} \right] \tilde{H}_{\mu\nu}. \quad (3.36)$$

For $\lambda = 1$, the result in the appendix is recovered. For the moment, let us keep λ arbitrary. Since (3.36) is of the form of (3.32), therefore it can be cancelled with $\delta_{(2)}B_{\mu\nu}$ if $\phi_{\mu\nu\kappa}$ satisfies

$$\partial_5\phi_{\mu\nu\kappa} - \frac{1}{2}\epsilon_{\mu\nu}^{\alpha\beta\gamma}D_\alpha\phi_{\beta\gamma\kappa} = -\frac{\lambda}{2}x_5\epsilon^{\mu\nu\alpha\beta\gamma}D_\alpha H_{\beta\gamma\kappa} - \frac{\lambda-1}{4}\tilde{H}^{\alpha\beta}\epsilon_{\kappa\alpha\beta\mu\nu} := J_{\mu\nu\kappa}. \quad (3.37)$$

In addition, we impose the boundary condition

$$\phi_{\mu\nu\kappa} \text{ vanishes as } |x_5| \rightarrow \infty. \quad (3.38)$$

A solution can always be written down using the Green function technique for general $J_{\mu\nu\kappa}$. Let $G_{\mu\nu,\mu'\nu'}^{ab}(x, y)$ be the Green function which satisfies

$$\partial_5 G_{\mu\nu}^{ab\mu'\nu'} - \frac{1}{2}\epsilon_{\mu\nu}^{\alpha\beta\gamma}(D_\alpha^{(y)})^a{}_c G_{\beta\gamma}^{cb\mu'\nu'} = \delta_{\mu\nu}^{\mu'\nu'} \delta^{ab} \delta^{(6)}(x - y) \quad (3.39)$$

and the boundary condition

$$G_{\mu\nu}^{ab\mu'\nu'}(x, y) = 0, \quad |x_5| \rightarrow \infty. \quad (3.40)$$

Here $x = (x^M)$ and $(D_\alpha)^a{}_c = \partial_\alpha\delta^a{}_c + (\tilde{A}_\alpha)^a{}_c$ where $(\tilde{A}_\alpha)^{ac} := f^{abc}A_\alpha^b$. Then

$$\phi_{\mu\nu\kappa}^a = \int dy G_{\mu\nu}^{ab\mu'\nu'}(x, y) J_{\mu'\nu'\kappa}^b(y) \quad (3.41)$$

satisfies both (3.37) and (3.38). As a result, if also

$$\delta A_\mu = 0, \quad (3.42)$$

then S_0 is invariant. So far this works for any λ .

Next let us examine the action S_E . It follows from (3.35) that

$$\delta\tilde{H}_{\mu\nu} = \partial_5\phi_{\mu\nu\kappa}\Lambda^\kappa + \frac{\Lambda \cdot x}{2}\epsilon_{\mu\nu}^{\alpha\beta\gamma}D_\alpha\tilde{H}_{\beta\gamma} + \frac{\lambda+1}{4}\epsilon_{\mu\nu}^{\alpha\beta\gamma}\Lambda_\alpha\tilde{H}_{\beta\gamma}, \quad (3.43)$$

where we have used the differential equation (3.37). Therefore S_E is invariant if we take $\lambda = -1$ and if $E_{\mu\nu}$ transforms as

$$\delta E_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}^{\alpha\beta\gamma}D_\alpha((\Lambda \cdot x)E_{\beta\gamma}). \quad (3.44)$$

All in all, our action is invariant under the transformation (3.35), (3.42) and (3.44).

In general the Lorentz invariance of the action implies that the equations of motion (i.e. (3.12), (3.24) (3.23) and (3.29)) are automatically Lorentz invariant, up to terms vanishes on shell and terms that can be interpreted as any other symmetry transformations of the theory. However since the self-duality equation (3.27) is obtained by a gauge fixing, it is not guaranteed to be Lorentz invariant. In fact, the transformation (3.35) implies that

$$\delta(\tilde{H}_{\mu\nu} - \partial_5 B_{\mu\nu}) = \frac{\Lambda \cdot x}{2} \epsilon_{\mu\nu}^{\alpha\beta\gamma} D_\alpha \tilde{H}_{\beta\gamma} - (\Lambda \cdot x) \partial_5 \tilde{H}_{\mu\nu} - \partial_5(x_5 H_{\mu\nu\kappa} \Lambda^\kappa). \quad (3.45)$$

This gives in (3.32) $\delta S_0 = 0$ as expected. Using the self-duality equation (3.27), the first and second term of (3.45) actually cancel and so

$$\delta(\tilde{H}_{\mu\nu} - \partial_5 B_{\mu\nu}) = -\partial_5(x_5 H_{\mu\nu\kappa} \Lambda^\kappa) + \text{EOM}, \quad (3.46)$$

where EOM denotes terms vanish when the equation of motion (3.27) is used. One can rewrite this further by using the equation of motion and obtains

$$\delta(\tilde{H}_{\mu\nu} - \partial_5 B_{\mu\nu}) = \frac{1}{2} \epsilon_{\mu\nu\kappa}^{\alpha\beta} \Lambda^\kappa (\tilde{H}_{\alpha\beta} + 2x_5 \partial_5 \tilde{H}_{\alpha\beta}) + x_5 \Lambda^\kappa D_\kappa \tilde{H}_{\mu\nu} + D_{[\mu} \varphi_{\nu]} + \text{EOM}, \quad (3.47)$$

where $\varphi_\nu = x_5 \tilde{H}_{\nu\kappa} \Lambda^\kappa$. Now the first and second term on the RHS of (3.47) respectively gives zero when substituted into (3.32) and so they corresponds to symmetry transformations of the action S_0 ⁶. For the abelian case, the third term corresponds to the symmetry transformation $\delta B_{\mu\nu} = \partial_{[\mu} \alpha_{\nu]}$ of $B_{\mu\nu}$ and since S_E decouples from the theory, so we obtain that the self-duality equation is Lorentz invariant up to terms vanishes on shell and terms that correspond to a symmetry transformation of the theory. However the above analysis breaks down in the non-abelian case and so we conclude that the self-duality equation of motion is not Lorentz invariant. We emphasize that the loss of Lorentz invariance in (3.27) is simply because it is a gauge fixed equation of motion. This is not surprising. For example, Yang-Mills equation of motion in the Coulomb gauge is not Lorentz invariant. The use of the self-duality equation is important for obtaining the correct counting on the degrees of freedom in the theory. However the use of the ungauged version (3.24) may be useful for some other purposes, for example, supersymmetry.

If we compute the algebra of commutator $[\delta(\Lambda_\mu^{(1)}), \delta(\Lambda_\mu^{(2)})]$ for the physical field $B_{\mu\nu}$, we get the standard 5d Lorentz transformation plus an additional transformation. This additional transformation is quite complicated but is a symmetry of the action since we know already the action is invariant under the 5d Lorentz transformation and is

⁶More specifically, the symmetry transformations are given by $\delta B_{\mu\nu} = \phi_{\mu\nu\kappa} \Lambda^\kappa$ where $\phi_{\mu\nu\kappa}$ is given by (3.41) with $J_{\mu\nu\kappa}$ specified by the first and second term of the RHS of (3.47) respectively.

invariant under $[\delta(\Lambda_\mu^{(1)}), \delta(\Lambda_\mu^{(2)})]$. Therefore we can interpret (3.35) as a modified Lorentz symmetry. Note that the form of the transformation laws (3.42) and (3.44) are quite non-standard but they are compatible with the auxiliary nature of these fields.

We note that as $\phi_{\mu\nu\kappa}$ is determined explicitly as an integrated expression over the Green function, the transformation (3.35) is non-local in the fields. It is now clear that the different choices of λ simply correspond to different non-local form of the transformation (3.35). What we have shown is that one can make the action invariant by using a transformation law that has a nonlocal piece that is based on a local part with the particular choice of $\lambda = -1$. For the abelian case, we know the Lorentz transformation (2.17) is locally represented in terms of A_μ and $B_{\mu\nu}$; and corresponds to $\lambda = 1$ and $\phi_{\mu\nu\kappa} = 0$. Let us demonstrate that this is equivalent to having $\lambda = -1$ and a nontrivial $\phi_{\mu\nu\kappa}$ as determined above. To see this, the equation (3.37) reduces in the abelian case to

$$\partial_5 \phi_{\mu\nu\kappa} - \frac{1}{2} \epsilon_{\mu\nu}^{\alpha\beta\gamma} \partial_\alpha \phi_{\beta\gamma\kappa} = x_5 \partial_\kappa \tilde{H}_{\mu\nu} - H_{\mu\nu\kappa}. \quad (3.48)$$

Let us put $\phi_{\mu\nu\kappa} = -2x_5 H_{\mu\nu\kappa} + \varphi_{\mu\nu\kappa}$ and so

$$\partial_5 \varphi_{\mu\nu\kappa} - \frac{1}{2} \epsilon_{\mu\nu}^{\alpha\beta\gamma} \partial_\alpha \varphi_{\beta\gamma\kappa} = -\frac{1}{2} \epsilon_{\mu\nu\kappa}^{\alpha\beta} (\tilde{H}_{\alpha\beta} + 2x_5 \partial_5 \tilde{H}_{\alpha\beta}) - x_5 \partial_\kappa \tilde{H}_{\mu\nu}. \quad (3.49)$$

Now the right hand side of this equation when substituted into (3.32) actually leaves S_0 invariant. Therefore as explained above, $\varphi_{\mu\nu\kappa}$ represents a symmetry and we recover (2.17) up to a symmetry transformation.

The Lorentz symmetry we proposed is nonlocal and is quite different from the usual representation of a symmetry in terms of local fields, but it seems this is what is needed for multiple M5-branes⁷. In fact, nonlocal symmetry is not uncommon in string theory. For example, the spacetime Lorentz symmetry in the light cone gauge string theory is nonlocal in the worldsheet coordinate [28]. There the nonlocality arises since a Lorentz transformation will generally bring one out of the lightcone gauge and so a worldsheet reparametrization (turns out to be nonlocal) is needed in order to restore the gauge condition. For us, we are in a formulation without the $B_{5\mu}$ fields. Since a standard 5- μ Lorentz transformation will turn $B_{\mu\nu}$ to $B_{5\mu}$, we suspect that the reason of having a modified Lorentz symmetry is similarly due to a compensating gauge transformation in a covariant formulation. In the abelian (free) case, the modification is not so drastic and the modified Lorentz transformation is still local. But this is not the case for the non-abelian case as we found here. To check our suspicion, it is needed to construct the covariantized theory. It is remarkable that for the abelian case, PST [5] were able to provide a Lorentz covariant formulation by introducing additional auxiliary fields (scalar field a and the $B_{5\mu}$ components). It will be very interesting to covariantize

⁷We thank Pei-Ming Ho and Yutaka Matsuo for emphasizing the nonlocal nature of our proposed Lorentz transformation and for a discussion on this point.

our construction by following a similar construction of PST and it is possible that the employment of additional auxiliary fields would allow for a local representation of the Lorentz symmetry.

3.2.3 Reduction to D4-Branes

Let us consider a compactification of x_5 on a circle of radius R . The dimensional reduced action reads

$$S = \frac{2\pi R}{2} \int d^5x \operatorname{tr} \left(-\tilde{H}_{\mu\nu}^2 + (F_{\mu\nu} - 2\pi R \tilde{H}_{\mu\nu}) E^{\mu\nu} \right) \quad (3.50)$$

This form of action has been considered in [10] as a dual formulation of 5-dimensional Yang-Mills theory. In fact, if we integrate out $E_{\mu\nu}$, we obtain the expected relation

$$F_{\mu\nu} = 2\pi R \tilde{H}_{\mu\nu}. \quad (3.51)$$

Eliminate $\tilde{H}_{\mu\nu}$ using the constraint, we obtain the standard 5d Yang-Mills action

$$S_{YM} = -\frac{1}{4\pi R} \int d^5x \operatorname{tr} F_{\mu\nu}^2. \quad (3.52)$$

This is however not the complete answer. In fact if we look at the path integral and integrate out E first, we obtain

$$\int [DA][DB][DE] e^{-S} = \int [DA][DB] e^{-S_{YM}} \delta(F_{\mu\nu} - 2\pi R \tilde{H}_{\mu\nu}) = \int [DA] e^{-S_{YM} - S'}, \quad (3.53)$$

where $S' = S'(A)$ is a measure contribution obtained from integrating out the delta functional constraint and then rewritten in terms of A_μ . The direct determination of S' is nontrivial but it has to satisfy a consistency condition: the condition

$$D_\mu F^{\mu\nu} = -\frac{\pi R}{2} \epsilon^{\nu\alpha\beta\gamma\delta} [F_{\alpha\beta}, B_{\gamma\delta}] \quad (3.54)$$

which follows from (3.51) should be obtained as an equation of motion in the 5d theory. As a result, S' has to satisfy

$$\frac{\delta S'}{\delta A_\nu} = \frac{1}{2} \epsilon^{\nu\alpha\beta\gamma\delta} [F_{\alpha\beta}, B_{\gamma\delta}] \quad (3.55)$$

with $B_{\mu\nu}$ understood to be a function of A_μ obtained by solving the duality relation (3.51).

The 5d theory is thus given by the action $S_{5d} = S_{YM} + S'$. The action S_{YM} corresponds to the expected form of the Yang-Mills coupling

$$g_{YM}^2 = R \quad (3.56)$$

and the gauge group in our construction is to be

$$G = U(N) \quad (3.57)$$

for a system of N M5-branes. The reproduction of the 5d Yang-Mills action gives further support that our construction gives a description of the gauge sector of a system of multiple M5-branes. The action S' describes a correction term to the Yang-Mills theory which appears to be of high derivative in nature since $[F, B] \sim DDB$ and B is of the order of F from (3.51)). In the abelian case, Perry and Schwarz has also constructed the nonlinear five-brane action that gives the $U(1)$ DBI action of D4-brane upon dimensional reduction. It would be interesting to work out S' in more details and see whether it captures the non-abelian DBI action [29] in some way.

We remark that the necessity of non-locality in the M5-branes action has also been argued by Witten [30]. He observed that conformal invariance of the M5-branes theory implies that upon double dimensional reduction to five dimensions, the 5 dimensional action should be proportional to

$$\frac{1}{R} \int d^5x. \quad (3.58)$$

On the other hand, one should get

$$\int d^6x = 2\pi R \int d^5x \quad (3.59)$$

as a result of integrating over the x_5 direction for a standard reduction of a local action, In our analysis above, we see that both R -dependence are correct and the trick to arrive from (3.58) to (3.59) is due to the simple R dependence in the constraint (3.51).

In principle one could consider compactification in the other spacelike directions and one should get the same 5d YM action. However this is already non-trivial for the Perry-Schwarz action [3] (or the Henneaux-Teitelboim action [11]) and implies the existence of a symmetry of the D4-branes action which involves a non-local field redefinition. For a single M5-brane, this symmetry can be made explicit in a covariant PST-like formulation in which both, the vector field A_μ and the two-form field $B_{\mu\nu}$ are present and related to each other, on the mass-shell, by the duality condition which follows from the action. See for example [31] for the case of the duality-symmetric formulation of $D = 11$ supergravity with A_3 and A_6 gauge fields. The construction is completely generic and can be extended immediately to arbitrary D dimensional spacetime any pair of duality related fields of rank p and $(D - p - 2)$ whose field strengths are dual to each other on the mass shell⁸. It would be interesting to extend this construction to the non-abelian case.

⁸We thank Dmitri Sorokin for explaining this to us.

4. Discussions

In this paper, we have constructed a theory of non-abelian tensor fields with the properties that:

1. the action admits a self-duality equation of motion,
2. the action has manifest 5d Lorentz symmetry and a modified 6d Lorentz symmetry,
3. on dimensional reduction, the action gives the 5d Yang-Mills action plus certain higher derivative corrections.

Based on these properties, we propose our action to be the bosonic theory describing the gauge sector of coincident M5-branes in flat space. A special feature of our construction is that the tensor gauge symmetry is abelian although the theory is still fully interacting. This is an interesting difference between the self-interaction of Yang-Mills gauge fields and the self-interaction of 2-form gauge fields in our construction. It remains to be seen whether this is still the case in the Lorentz covariant formulation of the theory.

We note that conformal symmetry rules out the possibility of a Yang-Mills action, but a 5d Chern-Simons action is allowed for the gauge field A_μ :

$$S_{CS} = \frac{k}{24\pi^2} \int d^5x \epsilon^{\mu_1 \dots \mu_5} \text{tr} \left(A_{\mu_1} \partial_{\mu_2} A_{\mu_3} \partial_{\mu_4} A_{\mu_5} + \frac{3}{2} A_{\mu_1} A_{\mu_2} A_{\mu_3} \partial_{\mu_4} A_{\mu_5} + \frac{3}{5} A_{\mu_1} A_{\mu_2} A_{\mu_3} A_{\mu_4} A_{\mu_5} \right). \quad (4.1)$$

The inclusion of the Chern-Simons action seems to corresponds to a kind of M-theory compactification as 5d Chern-Simons term naturally arises and plays a very important role in certain kinds of M-theory compactification on Calabi-Yau manifolds, see for example [32], [33]. In this case, the level k may corresponds to a parameter describing a kind of fibered Calabi-Yau compactification. It will certainly be helpful to have the full supersymmetric theory from which one may obtain the moduli space interpretation from the scalar sector [34].

Our construction is in principle only a low energy effective description for a system of coincident M5-branes. If one is lucky, the (2,0) supersymmetric completion may give a well-defined quantum theory as in the case of BLG [35] and ABJM theories [9] for multiple M2-branes and the $\mathcal{N} = 4$ SYM theory for multiple D3-branes. This is another strong reason to construct the supersymmetric completion.

To construct the supersymmetric theory, one needs to include scalar fields and fermions in the adjoint of $U(N)$. For (2,0) supersymmetry, all these fields are sitting in the tensor multiplet. Since there is no Yang-Mills multiplet in (2,0) supersymmetry, the Yang-Mills gauge field must be a supersymmetric singlet. This is rather difficult to

implement. On the other hand, it is possible that only a fraction of the (2,0) supersymmetry, i.e. (1,0) supersymmetry, is visible in the classical action of multiple M5-branes, and full supersymmetry can be seen only nonperturbatively as in the ABJM theory [9]. With respect to (1,0) supersymmetry, the (2,0) tensor multiplet is simply the sum of a (1,0) tensor multiplet and a (1,0) hyper-multiplet. Moreover, one should employ a (1,0) Yang-Mills multiplet as an auxiliary multiplet. The recent results of (1,0) superconformal theories [17] should be useful in this regard.

However even before one enters into the details, a simple observation already indicates that the supersymmetric theory is going to be highly nontrivial. In six dimensions, scalar field has dimension 2. Conformal invariance plus locality imply that the potential term V for the scalar fields has to be cubic. However a nonvanishing cubic potential has no ground state and this is not compatible with supersymmetry ⁹. This means the potential term, if nonvanishing, will need to be nonlocal. For example, potential of the schematic form $V \sim \phi^4/|\phi|$ or $V \sim \int dx_5 \int dx_5 \phi^4$ could avoid the problem of not having a ground state. It is amusing that the later form of the potential has a close resemblance with the scalar interaction term in [15] ¹⁰ if one exchanges $C_\mu \sim \delta_\mu^5 \int dx_5$, both of which are of dimension -1.

It would be interesting to understand the connection between our description and the proposed SYM description of M5-branes [13, 14]. In particular an understanding of how a non-abelian 2-form gauge field would arise in the Yang-Mills description is needed. Incidentally, based on a fluctuation analysis of D1-branes around a large RR 3-form flux background, a matrix model description for M5-branes in a background C -field was suggested in [19] and there is the same question of how to extract a B -field from the matrix variables. This problem may be compared with the problem of extracting the spacetime fields and their dynamics, particularly the gravity field, from the matrix model [36,37]. See for example [38–40]. Lessons drawn from those analysis may be useful here.

Our theory is based on fields in the adjoint of $U(N)$, i.e. taking N^2 values. Naively this is different from the N^3 counting from entropy argument [41]. To understand the counting, it will be important to understand the dynamics of the theory properly. See for example [22] for some recent interesting analysis performed on the 5d SYM theory and a class of 6d SCFT in the Coulomb phase.

A. Counting of degrees of freedom in the Perry-Schwarz theory

We give a pedagogical and explicit counting of the degrees of freedom in the Perry-

⁹This observation is also shared independently by David Berman, Neil Lambert, David Tong.

¹⁰We thank Neil Lambert for pointing out this resemblance.

Schwarz theory. The Perry-Schwarz theory initially has the equation of motion

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0 \quad (\text{A.1})$$

Using the gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu, \quad (\text{A.2})$$

one can fix the equation of motion to the linear form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}. \quad (\text{A.3})$$

Doing so we are left with a x^5 -independent residual symmetry. Now $\partial^\mu B_{\mu\nu}$ is x_5 independent as a result of (A.3). Using the residual symmetry, one can fix it to be zero

$$\partial^\mu B_{\mu\nu} = 0. \quad (\text{A.4})$$

Differentiating (A.3) with respect to x_5 and use (A.4), we obtain that $B_{\mu\nu}$ is massless as expected, $\square B_{\mu\nu} = 0$. Now (A.4) gives 4 independent conditions on the 10 components of $B_{\mu\nu}$. Using the self-duality condition, we have in total $(10-4)/2 = 3$ degrees of freedom.

B. Variation of S_0 under Lorentz transformation

In this appendix, we show that the non-abelian Perry-Schwarz action

$$S_0 = \frac{1}{2} \int d^6x \text{tr} \left(-\tilde{H}^{\mu\nu} \tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu} \partial_5 B_{\mu\nu} \right), \quad (\text{B.1})$$

is not invariant under the straight-forward non-abelian generalization of the Lorentz transformation (2.17) (i.e. with $\phi_{\mu\nu\kappa} = 0$ in (3.35)):

$$\delta B_{\mu\nu} = (\Lambda \cdot x) \tilde{H}_{\mu\nu} - x_5 \Lambda^\kappa H_{\kappa\mu\nu}, \quad (\text{B.2})$$

$$\delta A_\mu = 0. \quad (\text{B.3})$$

It is

$$2\delta S_0 = \int \epsilon^{\mu\nu\rho\lambda\sigma} \text{tr} \left[\left(\underbrace{(\Lambda \cdot x) \tilde{H}_{\mu\nu}}_1 - \underbrace{x_5 \Lambda^\kappa H_{\kappa\mu\nu}}_2 \right) \left(\underbrace{D_\rho \tilde{H}_{\lambda\sigma}}_a - \underbrace{D_\rho \partial_5 B_{\lambda\sigma}}_b \right) \right]. \quad (\text{B.4})$$

The contributions are, respectively,

$$(1a) = -\frac{1}{2} \int \text{tr} (\epsilon^{\mu\nu\rho\lambda\sigma} \Lambda_\rho \tilde{H}_{\mu\nu} \tilde{H}_{\alpha\beta}) + \text{tot.}, \quad (\text{B.5})$$

$$(2b) = - \int \text{tr} (\epsilon^{\mu\nu\lambda\alpha\beta} x_5 \tilde{H}_{\alpha\beta} \partial_5 \tilde{H}_{\mu\nu} \Lambda_\lambda) = \frac{1}{2} \int \text{tr} (\epsilon^{\mu\nu\rho\lambda\sigma} \Lambda_\rho \tilde{H}_{\mu\nu} \tilde{H}_{\alpha\beta}) + \text{tot.}, \quad (\text{B.6})$$

$$(1b) = -2 \int (\Lambda \cdot x) \text{tr} (\tilde{H}_{\mu\nu} \partial_5 \tilde{H}^{\mu\nu}) = \text{tot.}, \quad (\text{B.7})$$

$$(2a) = \int 2x_5 \Lambda^\kappa \text{tr} (H_{\kappa\mu\nu} D_\rho H^{\mu\nu\rho}) = \int 2x_5 \Lambda^\kappa \text{tr} \left(\frac{1}{3} H^{\rho\mu\nu} D_{[\kappa} H_{\rho\mu\nu]} \right) + \text{tot.}, \quad (\text{B.8})$$

where tot. stands for total derivative terms and we have used

$$D_{[\kappa} H_{\rho\mu\nu]} = D_\kappa H_{\rho\mu\nu} - D_{[\rho} H_{\mu\nu]\kappa} \quad (\text{B.9})$$

in simplifying (2a). We see that (1a) cancels (2b). In the abelian case, the term (2a) is zero due to the vanishing Bianchi identity $\partial_{[\kappa} H_{\rho\mu\nu]} = 0$. This is not so for the non-abelian case and so S_0 is not invariant under (B.2). It is straightforward to see that S_0 is also not invariant under

$$\delta B_{\mu\nu} = (\Lambda \cdot x) \tilde{H}_{\mu\nu} - x_5 (\Lambda \cdot D) B_{\mu\nu}. \quad (\text{B.10})$$

C. Counting of degrees of freedom for Chern-Simons theory

We will start with a review of the counting of degrees of freedom for pure Chern-Simons theory performed in [25, 26]. Then we extend the analysis to the case where the Chern-Simons theory is coupled to a covariantly conserved current. The details of the counting is not important for our results. They are included here for completeness.

C.1 Pure Non-Abelian Chern-Simons theory

Consider the five dimensional (dimension $D = 2n + 1$, $n = 2$ here) Chern-Simons action

$$S_{\text{CS}} = \int_M \mathcal{L}_{\text{CS}}, \quad \text{with} \quad d\mathcal{L}_{\text{CS}} = g_{abc} F^a \wedge F^b \wedge F^c \quad (\text{C.1})$$

where g_{abc} is the symmetric invariant tensor of the gauge group and $a = 1, \dots, \mathcal{N}$ with \mathcal{N} being the dimension of the gauge group. The equation of motion

$$g_{aa_1a_2} F_{\mu_1\mu_2}^{a_1} F_{\mu_3\mu_4}^{a_2} \epsilon^{\mu_1\mu_2\mu_3\mu_4\lambda} = 0 \quad (\text{C.2})$$

can be decomposed into

$$\begin{cases} k_a \equiv g_{aa_1a_2} F_{i_1i_2}^{a_1} F_{i_3i_4}^{a_2} \epsilon^{i_1i_2i_3i_4} = 0, \\ k_a^i \equiv 4g_{aa_1a_2} F_{i_1i_2}^{a_1} F_{0i_3}^{a_2} \epsilon^{i_1i_2i_3i} = 0, \end{cases} \quad (\text{C.3})$$

where $\mu = (0, i)$ and $i = 1, \dots, 2n$. Introduce the "2nN \times 2nN matrix" $\Omega_{ab}^{ij} \equiv 4\epsilon^{iji_1i_2} g_{abc} F_{i_1i_2}^c$ ((b, j) as a collective index), we can rewrite the equations of motion in the compact form:

$$\begin{cases} k_a = \Omega_{ab}^{ij} F_{ij}^b = 0 \\ \Omega_{ab}^{ij} F_{0j}^b = 0 \end{cases} \quad (\text{C.4})$$

A simple identity

$$\delta_{[k}^i g^{abc} \epsilon^{i\ell mn} F_{\ell\ell}^b F_{mn]}^c = 0, \quad \Rightarrow \quad \Omega_{ab}^{ij} F_{kj}^b = \delta_k^i k_a \quad (\text{C.5})$$

shows that on the constraint surface $k_a = 0$, $(v_k)_j^b \equiv F_{kj}^b$ gives $2n$ null vectors to Ω_{ab}^{ij} . The non-invertibility of Ω is due to the existence of symmetry. In this case, the $2n$ null vectors F_{kj}^b generates the spatial diffeomorphism. In fact under diffeomorphism $\delta x^\mu = \eta^\mu$ of spacetime, the Chern-Simons theory is invariant with $\delta_\eta A_\mu^a = \mathcal{L}_\eta A_\mu^a$, or the improved diffeomorphism

$$\delta_\eta A_\mu^a = -\epsilon^\nu F_{\mu\nu}^a. \quad (\text{C.6})$$

In general, the rank of Ω depends on the properties of the invariant tensor g_{abc} , and the phase space location of the system. For example, at $F_{\mu\nu}^a = 0$, $\Omega_{ab}^{ij} = 0$ and has zero rank. In [25, 26], a *generic condition* on g_{abc} was introduced. g_{abc} is said to be *generic* if there exists solution F_{ij}^a on the surface $k_a = 0$ such that:

- (a) The matrix F_{kj}^b ((b, j) as row and k as column index) has the maximum rank $2n$ such that $\xi^k F_{kj}^b = 0$ implies $\xi^k = 0$, i.e. the $2n$ null vectors $(v_k)_j^b \equiv F_{kj}^b$ of Ω_{ab}^{ij} are linearly independent.
- (b) The matrix Ω_{ab}^{ij} has maximum rank compatible with (a), i.e. Ω_{ab}^{ij} has no other null vectors except $(v_k)_j^b$ and so has rank $2nN - 2n$

We remark that the presence of the null vectors of Ω on the surface $k_a = 0$ is due to the presence of spatial diffeomorphism $\delta x^i = \eta^i$, $i = 1, 2, 3, 4$. (under generic condition assumption, temporal diffeomorphism is not independent). If there were no such diffeomorphism, we would not expect the existence of such null vectors.

Now the equation of motion (C.4) together with the generic condition implies $F_{0j}^b = N^k F_{kj}^b$ for arbitrary $2n$ fields N^k , or

$$\dot{A}_i^a = D_i A_0^a + N^k F_{ki}^a \quad (\text{C.7})$$

Since (C.7) is invariant under

- (a) Standard gauge transformation (\mathcal{N} dimensional) :

$$\delta A_i^a = -D_i \lambda^a, \quad \delta_\lambda A_0^a = -\dot{\lambda}^a - [\lambda, A_0]^a, \quad \delta_\lambda N^k = 0 \quad (\text{C.8})$$

- (b) Spatial diffeomorphism ($2n$ dimensional) :

$$\delta_\xi A_i^a = -\xi^j F_{ij}^a, \quad \delta_\xi A_0^a = -\xi^j F_{0j}^a, \quad \delta_\xi N^k = \dot{\xi}^k + [\xi, N]^k \quad (\text{C.9})$$

where $[\xi, N]^k$ is the Lie bracket of the vectors ξ and N ,

we can use the above symmetries to go to the the time gauge

$$A_0 = 0, \quad N^k = 0. \quad (\text{C.10})$$

In this case, the equation of motion is equivalent to

$$k_a = 0, \quad A_i^a = \text{time independent.} \quad (\text{C.11})$$

In addition to the \mathcal{N} constraints $k_a = 0$, the $2n\mathcal{N}$ functions $A_i^a(x_i)$ are subjected to the residual symmetry of the time gauge, these are \mathcal{N} time-independent gauge symmetry (C.8) as well as the $2n$ time-independent spatial diffeomorphism (C.9), therefore the number of arbitrary functions in the solution to the equation of motion of Lagrange formulation is $2n\mathcal{N} - \mathcal{N} - (\mathcal{N} + 2n) = 2(n\mathcal{N} - \mathcal{N} - n)$. The local degrees of freedom is simply the half of it, therefore

$$\text{no. of local degrees of freedom of pure CS} = n\mathcal{N} - \mathcal{N} - n \quad (\text{C.12})$$

with $n > 1$. In 5d, this would be $\mathcal{N} - 2$. We remark that the above analysis holds only for the non-abelian case. For the counting of local degrees of freedom in the abelian case, see [25, 26].

C.2 Chern-Simons theory coupled to conserved current

For the case that the Chern-Simons theory is coupled to a conserved current J^λ ($D_\lambda J^\lambda = 0$):

$$S = \int d^5x \text{tr} A_\mu J_\mu + S_{\text{CS}}, \quad (\text{C.13})$$

the equation of motion of A_λ is

$$g_{aa_1a_2} F_{\mu\nu}^{a_1} F_{\lambda\sigma}^{a_2} \epsilon^{\mu\nu\lambda\sigma\rho} = c J_\rho^a \quad (\text{C.14})$$

where c is some constant. In terms of the matrix $\Omega_{ab}^{ij} \equiv \epsilon^{iji_1i_2} g_{abc} F_{i_1i_2}^c$, the equation of motion can be written as

$$\begin{cases} \Omega_{ab}^{ij} F_{ij}^b = c J_0^a \\ 4\Omega_{ab}^{ij} F_{0j}^b = c J_i^a \end{cases} \quad (\text{C.15})$$

Generically, $J_i^a \neq 0$, this means that (C.5) can no longer be used to reduce the rank of Ω , so we have full rank $2n\mathcal{N}$ for Ω generically, i.e. Ω is invertible.

Now in the gauge $A_0^a = 0$, the second line of the equation of motion (C.15) simply provides a first order partial differential equation in time:

$$\partial_0 A_j^b = c(\Omega^{-1})_{ji}^{ab} J_i^a. \quad (\text{C.16})$$

As for the first equation of motion of (C.15), it is indeed time-independent since

$$\begin{aligned} \partial_0(\Omega_{ab}^{ij} F_{ij}^b - c J_0^a) &= \left(2g_{abc}\partial_0 F_{k\ell}^b F_{ij}^c \epsilon^{ijkl} - c\partial_0 J_0^a\right) \\ &= D_k[4g_{abc} F_{ij}^b F_{0\ell}^c \epsilon^{ijkl}] - c D_i J_i^a = c D_k J_k^a - c D_k J_k^a = 0 \end{aligned} \quad (\text{C.17})$$

As a result, (C.15) simply provides a constraint on the initial values $A_j^b(x_i, t = 0)$. Therefore, in the time gauge, $A_j^b(x_i, t)$ are determined by (C.16) up to the initial conditions $A_j^b(x_i, t = 0)$. Both the time-independent gauge transformation and the time-independent constraints (C.15) remove \mathcal{N} independent initial conditions, so we have local degrees of freedom

$$\frac{1}{2}(2n\mathcal{N} - \mathcal{N} - \mathcal{N}) = (n - 1)\mathcal{N} \quad (\text{C.18})$$

In 5d, it's \mathcal{N} .

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